

Perturbative Anomalies of the M–5–Brane

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ABSTRACT: We discuss several mechanisms to cancel the anomalies of a 5–brane embedded in M–theory. Two of them work, provided we impose suitable conditions either on the 11–dimensional manifold of M–theory or on the 4–form field strength of M–theory.

KEYWORDS: Branes in String Theory, M- and F-theories and Other Generalizations, Anomalies in Field and String Theories.

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1 Introduction

The world–volume massless content of the M theory 5–brane fills up the tensor multiplet of $(2, 0)$ supersymmetry in 6D. It contains 8 chiral spinors, 5 bosons which represent the directions transverse to the 5–brane and 3 components of an antiselfdual two–form. So the corresponding theory is potentially anomalous. The actual calculation shows that the chiral anomaly always vanishes when the 5–brane is considered in isolation, but does not in general vanish when the 5–brane is embedded into the M theory. Geometrically this corresponds to having a six–dimensional manifold W , the world–volume of the 5–brane, embedded in the eleven–dimensional manifold Q of M theory, (we are closely following the analysis and notations of [1]).

The anomalies in question arise from a breakdown of invariance under the diffeomorphisms of Q that preserve the embedding of W in Q . More precisely, the tangent bundle TQ of Q restricted to W decomposes according to $TQ|_W = TW \oplus N$, where TW and N are the tangent bundle and the normal bundle of W , respectively. A Riemannian metric on Q induces a Riemannian metric on W and a metric and an $SO(5)$ connection on N . The relevant diffeomorphisms of Q are those that map W to W . Any such diffeomorphism generates a diffeomorphism of W ; if the diffeomorphism induced on W is the identity, it determines a gauge transformation in N . Therefore the anomalies to be considered are the usual gravity anomalies in TW and the gauge anomalies in N . Their evaluation has been carried out in [2, 1]: the contributions from the fermions, the chiral two–form and the inflow from the bulk do not cancel completely, and the overall chiral anomaly has a

remarkably simple expression: it is generated via the usual descent equations from the 8-form

$$\begin{aligned} \frac{1}{24}p_2(N) &= d\omega_7(N), \\ 0 &= \delta\omega_7(N) + d\mathcal{A}_6, \end{aligned} \tag{1.1}$$

where $p_2(N)$ is the second Pontrjagin class of the normal bundle of the 5-brane world-volume and, for later use, we have introduced the corresponding properly normalized Chern–Simons term and 6-form anomaly $\omega_7(N)$ and \mathcal{A}_6 , respectively. Here δ represents the gauge transformations induced on the normal bundle.

It is shown in [1] that this residual anomaly can be canceled by means of a suitable 6D counterterm in the case in which Q is the product of a ten dimensional manifold and a circle, i.e. when M-theory reduces to type IIA superstring theory (in the limit of small radius) and the 5-brane reduces to the solitonic 5-brane of type IIA. Here we want to discuss the problem in general. The result of our investigation is that there are mechanisms to cancel the residual anomaly. However they work only at some price, which consists of imposing some conditions either on the topology of Q or on F_4 , the 4-form field strength of M-theory. In section 2, we determine two such conditions. Starting from the discussion of a topological counterterm, we show how to characterize Q so that this counterterm be well defined. We can alternatively construct a world-volume counterterm that cancels the anomaly, using the fact that a 5-brane is *framed*, see below. This works provided an additional condition on F_4 is imposed. An important problem, which we do not discuss in this paper, concerns the physical (or unphysical) nature of the framing.

To complete our discussion, in section 3 we have collected other cancellation mechanisms which, in our opinion, are not admissible or do not work.

Before we start our analysis, let us recall a few basic facts about anomalies. An anomaly, being the result of a local calculation, must be represented by a global basic form, therefore in eq. (1.1), $\omega_7(N)$ must also be a basic form. A global basic form $\omega_7(N)$ in W can be written by introducing a reference connection. Let A be the generic connection (with curvature F) in the principal bundle $P(W, SO(5))$ to which the normal bundle is associated, and A_0 the reference connection. Then

$$\begin{aligned} \omega_7(N) &= 4 \int_0^1 dt P_4(A - A_0, \mathcal{F}_t, \mathcal{F}_t, \mathcal{F}_t) \\ &= CS(A) - CS(A_0) + dS(A, A_0), \end{aligned} \tag{1.2}$$

where

$$CS(A) = 4 \int_0^1 dt P_4(A, F_t, F_t, F_t) \tag{1.3}$$

and $S(A, A_0)$ is a suitable 6-form in P (see [3]). Here \mathcal{F}_t is the curvature of $tA + (1 - t)A_0$ and $F_t = tdA + t^2/2[A, A]$. P_4 is the fourth order symmetric $SO(5)$ -ad-invariant polynomial corresponding to $p_2(N)/24$,

$$\frac{1}{24}p_2(N) = P_4(F^4). \tag{1.4}$$

The three forms in the RHS of (1.2) are all defined in the total space of $P(W, SO(5))$, while the LHS is basic.

We remark in particular that ω_7 being basic means that it vanishes identically in W , as W is a 6-dimensional space. This fact is immaterial as long as we deal with descent equations (for example, in section 2 of [4] it is shown how to reconcile it with the descent equations), but becomes very important if one wants to use $\omega_7(N)$ to construct an action term. This will be one of our main concerns in this paper.

2 Mechanisms for the anomaly cancellation

Let us start by recalling that a 5-brane magnetically couples to M-theory via the equation [5],

$$dF_4 = \delta_W \tag{2.1}$$

where F_4 is the 4-form field strength of M-theory, and δ_W is a closed form which represents the Poincarè dual of the world-volume manifold W (see also below). As long as (2.1) is a defining equation for the 5-brane, it implies, by Poincarè duality, that W is the boundary of some seven-dimensional manifold Y in Q ,

$$W = \partial Y. \tag{2.2}$$

In the following we are going to exploit this fact in order to cancel the residual anomaly. We are going to study two mechanisms.

2.1 A topological counterterm

We examine the possibility to cancel the anomaly of the M-5-brane by means of the counterterm,

$$S_{top} = \int_Y \omega_7 \tag{2.3}$$

where eq. (2.2) is understood. Formally,

$$\delta S_{top} = - \int_Y d\mathcal{A}_6 = - \int_W \mathcal{A}_6.$$

However we must exert some care in order for this to make sense. We have just recalled that $\omega_7(N)$, as defined in (1.1), is a basic form in W , and so it vanishes identically. Therefore in order to have a non-trivial counterterm, we must extend $\omega_7(N)$ to Y in a non-canonical (constructive) way, because the only canonical extension of 0 is 0; this in turn requires that we extend the connection and, before that, the normal bundle.

In summary, we must make sure that:

1. the normal bundle over W extends to a bundle over Y ;
2. the connection on the normal bundle extends too;

3. the counterterm (2.3) is independent of the choice of Y .

Conditions 1 and 2 are needed to ensure that the counterterm (2.3) makes sense. Let us analyze them first.

To this end the normal bundles need a more precise notation: $N(X, Z)$ will denote the normal bundle of X embedded in Z . For instance, according to this new notation, $N \equiv N(W, Q)$. First we have

$$i^*TQ \equiv TQ|_W = TW \oplus N(W, Q), \quad (2.4)$$

where $i: W \rightarrow Q$ is the embedding and

$$TQ|_W = TY|_W \oplus N(Y, Q).$$

As for Y , it is not closed, still we have

$$TY|_W = TW \oplus N(W, Y).$$

It follows that

$$N(W, Q) = N(W, Y) \oplus N(Y, Q)|_W.$$

Notice that $L \equiv N(W, Y)$ is a one-dimensional bundle. Since it is orientable, it is trivial and so we can write

$$N = N' \oplus L, \quad N' \equiv N(Y, Q)|_W, \quad L = W \times \mathbf{R}, \quad (2.5)$$

where N' is an $SO(4)$ -bundle, i.e. the gauge group $SO(5)$ reduces to $SO(4)$. The decomposition (2.5) implies in particular that $p_2(N) = p_2(N')$. Since

$$p_2(N') = p_2(N(Y, Q))|_W,$$

$p_2(N)$ extends to $0 \in H^8(Y)$. As forms we can write

$$p_2(N) = e(N(Y, Q))|_W^2,$$

where e denotes the Euler class.

After these preliminaries let us discuss the conditions 1 and 2 above. Using (2.5), one can trivially extend L and N' to bundles over Y as follows:

$$\begin{aligned} N' &= N(Y, Q)|_W \rightarrow N(Y, Q), \\ L &= W \times \mathbf{R} \rightarrow \tilde{L} = Y \times \mathbf{R}. \end{aligned}$$

Consequently N extends in a natural way as

$$N \rightarrow \tilde{N} = N(Y, Q) \oplus \tilde{L}. \quad (2.6)$$

We can now extend the connection over this bundle as follows. Let us construct a connection over \tilde{N} by taking the trivial connection in \tilde{L} plus the connection induced from

Q in $N(Y, Q)$. This is of course an extension of the connection induced from Q in N . Therefore also the form ω_7 extends, as well as all the operations on it. All this is made possible by (2.5).

It remains for us to discuss condition 3. By a standard argument the term (2.3) will not depend on the particular Y if

$$\frac{1}{2\pi} \int_X \omega_7 \in \mathbf{Z} \tag{2.7}$$

for any compact 7-manifold X without boundary. Looking at (1.2) it is actually easy to make a more precise statement about the integral in (2.7). We remark that ω_7 vanishes when $A = A_0$. Now, a generic connection A can be continuously joined to A_0 . Therefore also the results of the integral in (2.7) when evaluated at A_0 and at A should be continuously connected. This means that the only value of the integral compatible with (2.7) is *zero*.

This seems to be a very strong condition on the dynamics of the 5-brane, anyhow a hard to interpret one. Therefore we are prompted to take a different attitude: we consider for the time being unavoidable the idea of the action term (2.3) being dependent on Y .¹ With this attitude, a 5-brane in M-theory is characterized not only by the usual data (in particular by eq. (2.1)), but also by specifying a seven-manifold Y that bounds its world-volume. We hope a better understanding of M-theory, when available, will permit us to decide whether this Y -dependence is a physical characteristics of the 5-brane or a mere ‘gauge’ freedom.

However, if the 5-brane is to be specified by some other data beside the traditional ones, there is a less stringent and more manageable way to do it than by specifying a bounding manifold Y : we can simply add the specification of a ‘collar’ or ‘framing’ attached to the 5-brane. This procedure will be illustrated in the following subsection.

2.2 A world-volume counterterm

If the integral in (2.7) vanishes, it ensures the anomaly cancellation for any 5-brane embedded in Q . In this subsection we would like to explore a different point of view. We study anomaly cancellation conditions for a single 5-brane (not for all possible 5-branes) embedded in Q . The previous analysis told us that any 5-brane is *framed*, i.e. that its normal bundle splits, as in (2.5) – with rather non-standard terminology we call such a splitting a framing. If we consider a 5-brane with a *fixed* framing, we can construct a counterterm, akin to the one in [1], which, at some additional cost, cancels the anomaly. We stress again that the fixed framing in this section is part of the definition of the 5-brane and is a local property as opposed to the global existence of a bounding seven-manifold Y .

Therefore, let the normal bundle N split as

$$N = N' \oplus L, \tag{2.8}$$

¹We would like to point out that a similar problem of Y dependence appears in the attempts at writing a κ -symmetric action of the 5-brane in M-theory [6, 7]. Perhaps the two problems are connected.

where N' is an $SO(4)$ -bundle and L is a trivial line bundle. We have

$$p_2(N) = p_2(N') = e(N')^2 \quad (2.9)$$

and

$$\Phi(N) = pr_1^* \Phi(N') \wedge pr_2^* \Phi(L), \quad (2.10)$$

where pr_i , $i = 1, 2$ are the projections in the decomposition (2.8) of N and $\Phi(E)$ is the Thom class [8] of a real vector bundle E over W . Φ and e are related by

$$\sigma_0^* \Phi(N') = e(N'), \quad (2.11)$$

where $\sigma_0: W \rightarrow N'$ is the zero section of the bundle N' . From now on, for simplicity, we will drop the pull-back symbols.

Let $N_\epsilon(W) \subset N$ be a suitable tubular neighborhood of W of size ϵ embedded in Q . If we identify $N_\epsilon(W)$ with the corresponding submanifold of Q , then since the support of $\Phi(N)$ can be taken to lie inside $N_\epsilon(W)$, one can view $\Phi(N)$ as a form on $N_\epsilon(W)$ and identify it with the Poincaré dual of W , represented by the form δ_W .

Let ω_3 be the basic 3-form constructed out of the polynomial corresponding to $1/24e(N')$ just like ω_7 in eq. (1.2). Using the descent equations we get

$$\delta\omega_3 + da = 0,$$

(2.9) says that the integrated anomaly for the framed 5-brane is given by

$$\int_W \mathcal{A}_6 = \int_W e' \wedge a,$$

with $e' \equiv e(N')$.

Let v be a nonvanishing vertical vector field on L such that its contraction with $\Phi(L)$ satisfies

$$i_v \Phi(L)|_W = 1. \quad (2.12)$$

The existence of such a field follows from the 5-brane being framed. If s is the coordinate along the fiber of L , we can simply choose $v = \alpha \frac{\partial}{\partial s}$ with the constant α such that (2.12) be satisfied. If we now assume that the Lie derivative of F_4 in the L -direction is zero on W ,

$$\mathcal{L}_v F_4|_W = 0, \quad (2.13)$$

then, using (2.1), (2.10), (2.12) and (2.13), it is easy to see that

$$di_v F_4|_W = -e'. \quad (2.14)$$

Now, consider the counterterm

$$S = \int_W i_v F_4 \wedge \omega_3. \quad (2.15)$$

Its gauge variation is, using (2.14),

$$\delta S = - \int_W i_v F_4 \wedge da = \int_W di_v F_4 \wedge a = - \int_W e' \wedge a.$$

Hence the counterterm cancels the anomaly.

The case of IIA 5-brane, in which $Q = M \times S^1$, follows as a particular situation of the above. In this case the framing is determined by the background geometry.

Eq. (2.13), i.e. the constancy of F_4 along L on W , is the only condition required for this mechanism to work. It must be added to the defining eq. (2.1). We remark that the counterterm (2.15) depends on the vector field v , i.e. on the framing. In this section we have picked a definite one. However a 5-brane can have in general several distinct framings. In the previous subsection we saw that the 5-brane can have several distinct Y 's that bound it. Framings are local objects on the 5-brane, while the Y 's are non-local, and, if we cannot get rid of the Y dependence in (2.3), this may be a good reason to prefer the mechanism outlined in this subsection. Let us repeat again that the anomaly cancellation problem does not end here. The question of the physicality of v (or Y) remains open. To understand it a more complete formulation of M -theory with 5-branes would be extremely helpful.

3 Other mechanisms

3.1 A non-minimal 5-brane

The expression $p_2(N)/24$ is obtained via index theorem by summing the contribution from the chiral world-sheet fermions and antiself-dual two-form to the inflow anomaly. One possibility to cancel the anomaly is to consider a non-minimal 5-brane, i.e. to add $N = 2$ supermultiplets. These can be [9]:

- T =tensor (3,1;1)+(1,1;5)+(2,1;4)
- G =gravity (3,3;1)+(1,3;5)+(2,3;4)
- V =vector (3,2;1)+(1,2;5)+(2,2;4)

where the first two entries label the representations of the little group $SO(4) \sim SU(2) \times SU(2)$ and the third entry labels the representations of $USp(4) \sim SO(5)$, which we identify with the structure group of the normal bundle.

Then we use the relevant index theorems,

$$\begin{aligned} I_{(3,1;1)} &= -\frac{1}{8}L(TW), \\ I_{(2,1;4)} &= \frac{1}{2}\hat{A}(TW)chS_4(N), \\ I_{(1,3;5)} &= \frac{1}{8}L(M)ch_2S_5(N), \\ I_{(2,3;4)} &= -\frac{1}{2}\hat{A}(TW)(tr(e^{\frac{i}{2\pi}R}) - 1)chS_4(N), \end{aligned}$$

$$\begin{aligned}
I_{(3,2;1)} &= \frac{1}{2} \hat{A}(TW) (\text{tr}(e^{\frac{i}{2\pi}R}) - 1), \\
I_{(1,2;5)} &= -\frac{1}{2} \hat{A}(TW) \text{ch}S_5(N),
\end{aligned} \tag{3.1}$$

where $L(M)$ is the Hirzebruch polynomial, $\hat{A}(TW)$ the arithmetic genus of the tangent bundle, $\text{ch}V$ denotes the Chern character of the vector bundle V , R is the curvature of W , $S_n(N)$ denotes the spin bundle tensored with the vector bundle associated to the normal bundle via the representation \mathbf{n} of $SO(5)$ with $n = 4, 5$. Finally $\text{ch}_2V = \sum_{k=0}^{\infty} 2^k \text{ch}^{(k)}V$, where $\text{ch}^{(k)}V$ is the k -th order term in $\text{ch}V$, see for example [10].

We write the above contributions in terms of Pontrjagin classes $p_i(TW)$ and $p_i(N)$. For our purposes it will be enough to consider the irreducible contribution in terms of second Pontrjagin classes only. For the various multiplets they are as follows

$$\begin{aligned}
T : & \quad \frac{1}{48} (p_2(N) - p_2(TW)) \\
G : & \quad -\frac{21}{48} (p_2(N) - p_2(TW)) \\
V : & \quad \frac{4}{48} (p_2(N) - p_2(TW)).
\end{aligned} \tag{3.2}$$

Since the corresponding inflow anomaly contribution is proportional to $p_2(N) + p_2(TW)$, it is evident that by adding new multiplets it is not possible to cancel the anomaly.²

3.2 A generalized Green–Schwarz mechanism

A six–form potential C_6 couples naturally to the world–volume of the 5–brane. One therefore may wonder whether the term $\int_W C_6$ in the action could cancel the anomaly provided

$$\delta \int_W C_6 = - \int_W \mathcal{A}_6. \tag{3.3}$$

Following the analogy with the original Green–Schwarz mechanism, eq. (3.3) should come from

$$dF_7 = 1/24 p_2(N), \tag{3.4}$$

$$F_7 - dC_6 = \omega_7(N), \tag{3.5}$$

where the first equation is the modified Bianchi identity for the gauge invariant curvature F_7 of C_6 . However this fails in two respects.

First, the RHS of the above two equations vanishes. They are to be understood as equations in Q , but $p_2(N)$ and $\omega_7(N)$ are basic forms in W and thus vanish identically. The statement concerning $p_2(N)$ is obvious; as for $\omega_7(N)$ we have already explained this point in the introduction. The original Green–Schwarz mechanism involved 2–, 3– and 4–forms in a 10 dimensional space, and therefore the same conclusion was prevented. In

²The presence of G in the theory introduces anyhow new world–volume anomalies.

conclusion, the transformation (3.3) does not follow from the Bianchi identity. One might try to extend ω_7 to a non-vanishing 7-form in Q . However, as we explained in section 2, there may be many non-trivial extensions, but they are non-canonical and one has to specify them in a constructive manner, because any extension provides presumably different conditions either on Q or on the 5-brane embedding. An example of extension has been given in section 2.1 and we could repeat here a similar analysis. However it would be an academic exercise since there is a second objection.

The second reason is that in M-theory in the presence of a 5-brane, eq. (3.4) is not quite correct, but must be modified [5]. As an equation of motion of M-theory coupled to a 5-brane, it should contain other terms:

$$dF_7 + \frac{1}{2}F_4 \wedge F_4 - T \wedge \delta_W + I_8 = 0, \tag{3.6}$$

where F_7 is the Hodge dual of F_4 , T is the anti-self-dual 3-form on W satisfying [11] $dT = F_4|_W$ and is suitably extended to Q . I_8 is the 8-form responsible for the inflow anomaly.

Eq. (3.6) is consistent, but we do not see how eq. (3.3) can be derived from it. For this reason we discard also this cancellation mechanism.

3.3 A more exotic possibility

The analysis below grew out of an attempt to generalize a suggestion contained in [1]. Let us start with writing the residual anomaly in a form suitable for our discussion. To this end we introduce the 4×4 $SO(5)$ gamma matrices, γ_a , $a = 1, \dots, 5$, which satisfy

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab}, \quad \eta_{ab} = \text{diag}(1, 1, 1, 1, 1). \tag{3.7}$$

In particular we have

$$\text{Tr}(\gamma_a \gamma_b \gamma_c \gamma_d \gamma_e) = 4\epsilon_{abcde}, \quad \epsilon_{12345} = 1. \tag{3.8}$$

It is well-known that the generators of $SO(5)$, in the representation $\mathbf{4}$, can be written as

$$\Sigma_{ab} = -\frac{1}{4}[\gamma_a, \gamma_b]. \tag{3.9}$$

We write the connection and curvature in the normal bundle N as $A = A^{ab}\Sigma_{ab}$ and $F = F^{ab}\Sigma_{ab}$. Using this we find

$$p_2(N) = -\frac{1}{(2\pi)^{4 \cdot 4!}} \sum_{a=1}^5 \text{Tr}(\gamma_a FF) \text{Tr}(\gamma^a FF),$$

where the trace is over the gamma matrices, i.e. over the representation $\mathbf{4}$ of $SO(5)$. This is in explicit form the decomposition proposed in [1]. However we have not been able to implement the descent equations on a single factor $\chi_a = \text{Tr}(\gamma_a FF)$. For example, it is easy to see that there does not exist any combination ϕ_a of three forms $\text{Tr}(\gamma_a XY)$, where X, Y are either A, dA or $[A, A]$, such that

$$\chi_a = d\phi_a + wA_a{}^b\phi_b$$

for any weight w .

Trying to identify a non-abelian cancellation mechanism, by generalizing the above ideas, we were lead to a Green-Schwarz scheme, but, this time, with a non-abelian two-form potential. Non-Abelian tensor potentials have appeared recently in the literature [12, 13]. What follows will be very sketchy, we only present an example of the type of arguments we have used to exclude this mechanism too.

One can extract the explicit form of the anomaly from the eight-form (1.4) via the descent equations and get

$$\mathcal{A}_6 = 6 \int_0^1 dt (t - 1) P_4(dc, A, F_t, F_t), \tag{3.10}$$

where $c = c^{ab}\Sigma_{ab}$ is the gauge parameter.

To cancel this anomaly one may think of the following counterterm

$$\mathcal{C} = \int_0^1 dt \left\{ f P_4(\mathcal{B}, F_t, F_t) + g P_4(\mathcal{B}, (F_t, F_t)') + h P_4(\mathcal{B}, (F_t, F_t)'') \right\}. \tag{3.11}$$

Here f, g, h are t -dependent function to be determined and $'$ denotes a derivative with respect to t . The potential \mathcal{B} is a two-form that belongs to the symmetric part of $\mathbf{10} \otimes \mathbf{10}$ decomposition,

$$\mathbf{10} \otimes \mathbf{10} = \mathbf{35}_s + \mathbf{35}'_a + \mathbf{14}_s + \mathbf{10}_a + \mathbf{5}_s + \mathbf{1}_s, \tag{3.12}$$

i.e. it is a tensor $\mathcal{B}^{ab,cd}$ antisymmetric in $a \leftrightarrow b$ and $c \leftrightarrow d$ and symmetric in the exchange of ab with cd . A gauge transformation, $\delta A = dc + [A, c]$, is assumed to act on \mathcal{B} for example as follows

$$\delta \mathcal{B} = [\mathcal{B}, c] + \beta dcA. \tag{3.13}$$

In the last term we understand that we take the symmetric part of the tensor product between the Lie algebra generators. β is another t -dependent function to be determined. This implies in particular that \mathcal{B} is also t -dependent.

We now impose that

$$\delta \mathcal{C} = - \int_0^1 dt (t - 1) P_4(dc, A, F_t, F_t)$$

and work out the conditions for which this is satisfied. They consist of differential and algebraic equations on f, g, h, β , plus boundary conditions at $t = 0$ and $t = 1$. The system does not admit non-trivial solutions.

4 Comments

In section 2 we have presented cancellation schemes for the residual anomaly of the 5-brane embedded in M-theory. They work provided certain conditions are imposed either

on the topology of Q or on the dynamics of the 5-brane. These mechanisms may not be the only ones. For example, one could try a counterterm³

$$\int_Q F_4 \wedge \omega_7. \quad (4.1)$$

This term identically vanishes for the reasons explained in subsection 2.1 and 3.2. However one may try to extend ω_7 to a non-vanishing 7-form in Q . Our point of view should be clear by now: as explained above, this extension, if it exists, is non-canonical and must be explicitly constructed.

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³This counterterm is also studied in ref. [14]. We became aware of this reference after completion of our paper. We thank P.West for pointing it out to us. Section 7 of [14] also analyzes the 5-brane anomaly problem in M-theory; but there different cancellation mechanisms are considered, except for one case, corresponding to our section 3.3, where our conclusion is anyhow different.

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